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A topological approach to designing and constructing dynamical visual metaphors of multicultural and intercultural systems II-A

Una aproximación topológica al diseño y construcción de metáforas visuales dinámicas de sistemas multiculturales e interculturales II-A

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Resumen

En este ensayo continuamos nuestro trabajo previo en relación a intentar dotar a los estudios multiculturales e interculturales de un sólido fundamento topológico y de sistemas dinámicos. En primer lugar, nos aproximamos a las antiguas cosmovisiones taoístas y platónicas para mostrar que a pesar de sus diferencias, ambas comparte una estructura topológica común, a saber, $\{X, \Phi, A, B\}$, donde $A \cup B = X$ y $A \cap B = \Phi$. Más allá de argumentos metafísicos, este tipo de completitud disjunta es la estructura topológica básica que soporta al dualismo y a los abordajes dualistas de toda clase de fenómenos y problemas en ciencias y humanidades. El dualismo, además, conduce de forma natural a formular los modelos más sencillos de sistemas dinámicos multidimensionales no triviales. Luego de mostrar, a través de ejemplos y casos de estudio, que las topologías pueden ser pensadas como herramientas tanto de análisis como de diseño, procedemos a revisar algunos fundamentos de topología y sistemas dinámicos para darle soporte a la construcción de mapas de clasificación topológica para sistemas dinámicos de segundo orden, nuestra herramienta básica para modelar y clasificar los patrones de comportamiento cualitativos no equivalentes que un sistema dinámico dual, es decir, de segundo orden, puede exhibir. Hacia el final del ensayo regresamos al libro de leyendas de la Hermandad de los Monjes Azules y la Tribu de los Guerreros Escarlata, tomamos prestada de la biofísica la dinámica de las ecuaciones de Fitzhugh, y las utilizamos para mostrar todas las dinámicas genéricas locales que una tal sociedad bicultural Azul-Escarlata podría tener. Por razones de espacio este ensayo será publicado en dos partes. Esta es la primera de ellas.

Palabras claves: Matemáticas, Topología, Sistemas Dinámicos, Modelos, Metáforas, Multiculturalismo, Interculturalismo.

Abstract

In this work we continue our previous work on essaying giving multicultural and intercultural studies some sound topological and dynamical systems foundation. We first approach ancient Taoist and Platonist cosmovisions to show that despite their differences both share a common topology, namely, $\{X, \Phi, A, B\}$ with $A \cup B = X$, and $A \cap B = \Phi$. Metaphysical arguments aside, this kind of disjoint completeness is the basic topological structure supporting dualism and dualist approaches in every sort of phenomena and problems in sciences and humanities. Dualism, moreover, naturally leads to formulate the simplest models for nontrivial multidimensional dynamical systems. After showing, through examples and case studies, that topologies may be thought of both as analysis or design tools, we proceed to review some basics of topology and dynamical systems to support the construction of topological classification maps for second-order dynamical systems, our basic tool for modeling and classifying all non-equivalent qualitative patterns of behavior a dual, that is to say, a second-order, dynamical system, may exhibit. Towards the end of the essay we go back to the book of legends of the ancient Brotherhood of the Blue Monks and the Tribe of the Red Knights, borrow Fitzhugh equations dynamics from biophysics, and uses it to show all the local generic dynamics such a Blue-Red bicultural society might have. This paper will be published in two parts. This is the first one of them.

Keywords: Mathematics, Topology, Dynamical Systems, Models, Metaphors, Multiculturalism, Interculturalism.

1 Introduction

1.1 Ancient Contemporary Problems

For a twenty-first century man who has gone through a western education system, it is very hard to put aside his local western cultural myths, his technical backgrounds, and his already unconscious specialized languages, to approach problems pristinely, as one would suppose the antiques did long time ago, when they dealt with the very same problems for the first time. Most important contemporary problems are not at all new, and can in fact be documentary traced back to the most remote past, at the very beginning of history. Moreover, given that a few thousand years of written culture, and even less of documented mathematics, are less than an instant with respect to astronomical and biological scales of time, there is no sound rational reasons to think that our forefathers had not already been thinking and talking about the same problems for another thousands of years during prehistoric times, even though they did not leave behind traceable written bibliographical proofs of it. What makes old problems seem young is not their inner (geometric, algebraic, or material) structure but their appearance, perceived through, and therefore conditioned by, the lens of available technology. Technology, the set of both technical and conceptual tools at hand, among other things, also conditions and limits the possible approaches to problems, the expected results, and the kind of admissible solutions. Along this paper we will try to approach some problems like if we were citizens of ancient forgotten worlds, first at all to put into evidence that problems like multiculturalism and interculturalism are far from being just contemporary problems. Quite on the contrary, mono-, multi-, inter- and transculturalism are mathematical universals, in the sense that they are present not only in the study of sociological phenomena, but also in many different fields of knowledge, at different levels, and different epochs.

1.2 Duality and Topologies

The concept of duality, then that of multiplicity, hence the derived idea of multiculturality, is known since the origin of times. In fact, for the sake of establishing an eastern chronological landmark, we can explicitly found it in the Dao De Jing (Wang 2010) of Lao Zi, DDJ 2 1-10, the fundamental book of Taoists, written around two thousand two hundred years ago:

> "When the people of the world know the beautiful
> as beautiful,
> There arises the recognition of the ugly.
> When they know the good as good,
> There arises the recognition of the evil.
> This is the reason why
> Have-substance and have-no-substance produce each other;
>
> Difficult and easy complete each other;
> Long and short contrast with each other;
> High and low are distinguished from each other;
> Sound and voice harmonize with each other;
> Front and back follow each other." (DDJ 2 1-10)

Even tough dualism is a corner stone of Taoist cosmovision, it is nothing of an exclusive eastern concept or method of approaching reality, and we can find it in several antique western sources also. Plato (427-347 AC), a Greek contemporary of Lao Zi, is actually one of the canonical representatives of an ancient type of western dualism (Robinson 2017). In what to this paper is concerned, Plato's doctrine is summarized in his theory of "*ideas*" or "*forms*", in which the meaning of general words plays a key role. *General words* are labels used to identify eternal time-invariant universals, the *ideas* or *forms*, with no position in space or time. Yet, the concrete realizations of the ideas or forms are objects belonging to the physical world, to the world of the senses, and always constitute imperfect version of the absolute, pure forms. Bertrand Russell (Russell 1972) illustrates the logical aspects of Plato's theory of ideas contrasting the "idea", the "concept", of cat with an actual physical cat.

> "There are many individual animals of whom we can truly say "this is a cat". What do we mean by the word "cat"? Obviously something different from each particular cat. An animal is a cat, it would seem, because it participates in a general nature common to all cats. Language cannot get on without general words such as "cat", and such words are evidently not meaningless. But if the word "cat" means anything, it means something which is not this or that cat, but some kind of universal cattyness. This is not born when a particular cat is born, and does not die when it dies. In fact, it has no position in space or time; it is "eternal". (Russell 1972)

Even though Lao Zi and Plato belonged to distinct cultures and civilizations, and developed their "theories" independently of each other, in modern mathematical language, they could have formulated their respective cosmovisions using the same mathematical structure. In fact, let us denote the Taoist universe by T. Then, according to the poem DDJ 2 1-10 above, we could gather together the elements of T in two disjoint subsets, namely,

$$X = \{Beauty, Goodness, Have - substance, \text{Difficultness}, Longness, Highness, Sound, Front, \ldots\}, \quad (1)$$

$$Y = \{Ugliness, Evilness, Have - no - substance, Easiness, Shortness, Lowness, Voice, Back \ \ldots\}. \quad (2)$$

The sets X and Y are such that $X \cup Y = T$, and $X \cap Y = \Phi$. So, the set of subsets $\mathcal{T} = \{T, X, Y, \Phi\}$ defines a *topology* on T, and therefore the pair $(T, \mathcal{T})$ is a *topological space* (Munkres 1975, Dugundji 1976).■

Analogously, if P is the Platonist universe, $I = \{Ideas\}$, and $R = \{Realities\}$, then $I \cup R = P$, $I \cap R = \Phi$, the set of subsets $\mathcal{P} = \{P, I, R, \Phi\}$ defines a topology for P, and therefore the pair $(P,\ \mathcal{P})$ is also a topological space.■

An old mathematical saying asserts that the first time a mathematician uses an intuitive trick to solve a problem, the intuitive trick is … a trick. Yet, next time he uses the same intuitive trick to solve a different problem, the intuitive trick stops being a trick to become a method. Within this framework, the structural mathematical coincidence of the Taoist and the Platonist cosmovisions suggests that such a coincidence is not simply a coincidence but a method, and it seems to be so.

1.3 New Topologies for Old Worlds

From an engineering point of view, up until now we have approached dualism in Taoist and Platonist cosmovisions as an analysis problem, that is to say, dualist Taoist and Platonist cosmovision are external systems whose inner structures and laws we wish to understand and, were it possible, mathematically formulate. Yet, dualism could also be thought of as a design tool to induce desired dynamics or to control undesired dynamics in, for instance, social systems or historical processes. Western history can provide some additional examples.

Case Study 1. The War to the Death

In many senses it could be said that Venezuelan Independence War, the Violent Revolution, as John Lynch described it (Lynch 1986), started as a civil war in a province of the Spanish America, before evolving into an open Independence war between Venezuela and Spain. Very roughly we could say the independence war started soon after the independence was declared on 5 July 1811, in an atomized society of antagonist fractions: royalists, republicans, catholic clergymen, anti-clericalists, patriots, autonomists, independentists, radicals, conservatives, Spaniards, Americans, whites, creoles, pardos, zambos, mulatos, indians, free negros, slaves negros, among others, which depending on the circumstances could also change party. The liberation war of Venezuela was cruel, destructive and total since its beginning, reducing the population of the province from 800.000 in 1810 to approximately 700.000 in 1825, and the livestock (cattle, horses, mules) from 4.5 million head in 1812 to 256.000 in 1823 (Lynch 1986).

From the Venezuelan side of the conflict, the War to the Death run from 15 June 1813, when Bolívar signed the decree of War to the Death in Trujillo (Bolívar 1813), to 26 November 1820 when Bolívar and Morillo signed the armistice of Santa Ana, Trujillo, regularizing what had been an extermination war between Spain and Venezuela.

Topologically speaking, the decree of War to the Death of Bolívar introduced the dualist argument *Americans vs. Spaniards* into the Venezuelan liberation war:

> *"Any Spaniard who does not work against tyranny in favour of the just cause, by the most active and effective means, shall be considered an enemy and punished as a traitor to the country, and in consequence shall inevitably be shot … Spaniards and Canarios, depend upon it, you will die, even if you are simply neutral, unless you actively espouse the liberation of America. Americans, you will be spared, even when you are culpable" (Lynch 1986)*

pretending to transform a dynamic chaotic topology of small human factions in mutual conflicts, into a topology of two irreconcilable and mutually exclusionist parties: Venezuela against Spain. In Lynch's words:

> *"The Trujillo decree ruthlessly distinguished between Spaniards and Americans; it sought to cut through categories like royalism and republicanism and to make this a war between nations, between Spain and America. To this extent the decree of war to the death was an affirmation of Americanism, an expression of Venezuelan identity". (Lynch 1986).*

Had Bolívar succeeded on his war redesigning strategy, the universe U of the people in conflict would have been divided into only two subset, namely, $V = \{Venezuelans\}$ and $S = \{Spaniards\}$, satisfying the conditions: $V \cup S = U$, $V \cap S = \Phi$. So, the set of subsets of U, $\mathcal{U} = \{U, V, S, \Phi\}$, would define a topology on U, and therefore the pair $(U, \mathcal{U})$ would also be a topological space.■

The three cases above show that dualist cosmovisions or approaches may be naturally found both in analysis and synthesis problems in different contextual fields. Even tough dualism is certainly based on simplifications of structures and interactions, it provides the simplest possible models, either verbal or mathematical, for processes involving two or more actors or forces. Despite simplifications, and their eventual associated limitations, the historical success of dualist theories and models derive from the possibility of using them either as qualitative tools for thought, or quantitative tools for action. Qualitative and quantitative approaches are by themselves another example of complementary dualism: qualitative tools allow solving general classification problems, whereas quantitative tools permit computing particular behaviors with the required precision. In next sections we will come back to this problem.

If the success of dualism were associated to the simplicity of the topology associated to it, it would be reasonable to expect that destroying dualism would increase the complexity of the dynamics a system may exhibit. We will close this section with a second case study, chosen to illustrate the consequences of destroying duality in a socio-technological system. This example suggests that eroding local cultural myths, which according to Harari are responsible for keeping tighten together big human conglomerates (Harari 2017), is a necessary condition for implementing divide and conquer control strategies.

1.4 Collapsing and Exploding Topologies

The contemporary world has been imperceptibly dominated during last few decades by new cultural paradigms which structurally depend on the appropriate level of maintenance of different kinds of *networks* in optimal conditions: road and highway networks, massive transport networks, maritime transport networks, air transport networks, telecommunication networks, data transmission networks, radar networks, weather forecast networks, seller-purchaser networks, commerce networks, health and sanitation networks, education networks, cultural networks, spiritual networks, early alert networks, missile networks, defense networks, etc. Many of these networks are in turn interconnected, so that their appropriate functioning depends on assuring both the interconnectivity and the correct working of each one of them, and, at the end, they all depend on power and electricity networks. In developed countries people give for assured that all supporting networks run and will run smoothly, what use to be true, and this collective assumption acts as a universally shared local cultural myth (Harari 2017), with plenty of implications on people's life even if they are not aware of it. It is this very high level of certainty and confidence what assures that either mammoth companies like Amazon and Alibaba, with millions of customers around the globe, or small second-hand bookshops, with only a few hundred clients around the world, could essentially use the same protocols to run their business. To both Amazon and the small second-hand bookstore it is completely irrelevant who are you, or where you live, for instance. The only really relevant and important data for both sellers is whether you are connected to Internet or not; if you are connected you exist, and automatically start constituting a second-order dynamical system with the seller; if you are not connected you actually do not exist as a client, point.

Case Study 2. Automated Markets and Blackouts

The history of money precedes written history by thousands of years and has been a long and complex process (Wikipedia 2019), which we do not pretend to summarize here. We will limit ourselves to establish a couple of approximate dates, to fix chronological landmarks. Metal coins were independently invented around the seventh century B.C. in three of the ancient big cultures: China, India and Greece, and paper money was first introduce during the Song dynasty during the 11th century in China (Wikipedia 2019). Since then coins and bills coexisted and were used as the universal physical counterpart for buying and selling every kind of goods, either locally among people of small towns, and globally between nations. With the development of communication and data process networks virtual money has been progressively introduced worldwide, what has globalized the commercial interchange between people of developed countries. Technology notwithstanding, coins and notes keep playing a very important role in everyday life for most of humankind; it is simply still too early to completely deprive a developed society of circulating physical money. Yet what would never occur in a developed country may nonetheless happen, and is actually happening in Venezuela nowadays.

If today (18 September 2019) you go to a bank for cash in Mérida, Venezuela, you will get at most 6,000 "Bolívares soberanos", exactly enough to buy four pieces of "pan francés", (at 1,500 soberanos each), the smallest and cheapest piece of bread you can buy anywhere in the city. Yet, with 6,000 soberanos you could not pay a cup of coffee (24,000 soberanos per cup), and even less buy a pack of "Harina Pan" (maize flour at 25,000 soberanos per kg) for cooking "arepas" for dinner at home. These three examples just reveal what everybody knows: there is virtually no cash in Venezuela today.

What is the alternative to physical money? The generic contemporary alternative to cash is electronic commerce, but that depends on the appropriate functioning of power, telecommunications, and data networks, which unfortunately is neither the case in contemporary Venezuela, where most of the country suffers daily blackouts of between 6 and 8 hours, sometimes even twice a day.

What does all this have to do with topology and dynamical systems? Let us denote by X the set consisting of the seller S and the set C of his clients at the Principal Market in Mérida. Then $\{S\}\cup C = X$, $\{S\}\cap C = \Phi$, i.e., we assume S is not a client of himself, the set of subsets of X $\mathcal{C} = \{X, S, C, \Phi\}$ defines a topology for X, and $(C, \mathcal{C})$ is a topological space, which is essentially identical to the topological models of the other three examples already studied above.

Under ideal conditions, where the topology $\mathcal{C} = \{X, S, C, \Phi\}$ holds, the interrelationship between the seller S and his clients C could be modeled, in first instance, by a bilinear dynamical system (Arnol'd 1992, Hirsch and Smale 1974) like

$$\dot{S} = \alpha_S S - \beta_S S^2 + \gamma_S SC + \delta_S u_S(t) \tag{3}$$

$$\dot{C} = \alpha_C C - \beta_C C^2 + \gamma_C SC + \delta_C u_C(t), \tag{4}$$

where all the coefficients are supposed to be positive real numbers, S^2 and C^2 are the autoregulatory terms, SC models the interaction between the seller S and the client C, and the input signals $u_S(t)$ and $u_C(t)$ model the action of eventual external regulatory signals over the seller S and the clients C, respectively.

When a blackout occurs the set C of clients explodes into two main subsets, namely, the subset D of clients who can pay either in Venezuelan currency or foreign cash, and the subset G of the vast majority of clients whose debit cards are their unique payment tool. Yet, G may in turn be divided into two subsets: the subset E of the old regular well know client-friends of seller S, and F the subset of random unknown clients. So, in the presence of a blackout, the new configuration of the set of clients is

$$C = D \cup E \cup F, \tag{5}$$

$$D \cap E = \Phi, D \cap F = \Phi, E \cap F = \Phi. \quad (6)$$

The sets C, D, G, E, F are all subsets of X, and the set of sets $\mathfrak{X} = \{X, S, C, D, G, E, F, \Phi\}$ defines a topology for X, wherefrom $(X, \mathfrak{X})$ is also a topological space.

A second preliminary model for the dynamical system (S, C) that takes into account the explosion of the client subsets C during blackouts is:

$$\dot{S} = \alpha_S S - \beta_S S^2 + \gamma_s SD + \mu_S SE - \rho_S SF + \sigma_S u_S(t) \quad (7)$$

$$\dot{D} = \alpha_D D - \beta_D D^2 + \gamma_D DS + \delta_D u_D(t) \quad (8)$$

$$\dot{E} = \alpha_E E - \beta_E E^2 + \gamma_E ES + \delta_E u_E(t) \quad (9)$$

$$\dot{F} = \alpha_F F - \beta_F F^2 - \gamma_F FS + \delta_F u_F(t), \quad (10)$$

where, according to the signs of the right hand sides of equations (7-10), during blackouts the interaction between the seller S and the subsets of clients D and E keep being mutually beneficial, but the interactions with the subset of clients F, those whose debit cards stopped functioning, turns negative. Unfortunately, in the Venezuelan case, the subset F represents the vast majority of people, and they have really suffer a lot during day long blackouts because most shop, and very specially pharmacies, just closed their doors. Quantitatively speaking the subset of people who can pay in Venezuelan cash is just marginal, because the "Bolivar Soberano" is so weak and scarce that it is only used to pay for gasoline, and public transportation. During long lasting blackouts people started using foreign currencies to pay for food and medicines, but in practice only few people have access to foreign currency. According to Harari's Sapiens (Harari 2017) human relationship in small groups of people depends on mutual knowledge and confidence, and blackouts have proved this to be so in old small shops, where sellers and clients have known each other for a long time. In this case sellers have frequently allowed their old regular clients just to bring home what they needed, and clients have paid them after the blackouts. These mechanisms are bounded to groups of no more that 150 people (Harari 2017), but are worth to mention because they reflect structural properties of human groups. Nothing like that happened in big automated supermarkets; perhaps automation improves efficiency, but definitely destroys solidarity.

From the mathematical point of view it is interesting to observe that, within the context of dynamical systems, failures may be thought of as changes in the structure of the set of open set defining the topologies of the systems; such changes can be metaphorically thought of as explosions that break apart the open sets the topology of a system consists of into smaller subsets. The topology's explosions in turn induce associated explosions in the order of the dynamical systems, which increases accordingly. So, the finer the topology of the set of components of the dynamical system, the higher the order of the dynamical systems representation of the system, and the more complex the dynamics the system may exhibit. Within the context of failure theory in engineering systems, the increments in complexity associated to the explosions in the topology of the system might be completely undesirable; yet, creativity might also be thought of as a consequence of successive changes and refinements of the topological structure of the mind of every kind of creative people, from artists to mathematicians.

The sequence of images *A Topological Model of a Biblical Story* (Rodríguez-Millán 2017A) is a visual metaphor of the evolution of the universe during the first days of the creation according to the Genesis, thought of as a sequence of successive refinements of the topology of the primigenial universe (Rodríguez-Millán 2017B). A sample of this sequence of images is shown in Figure 1.

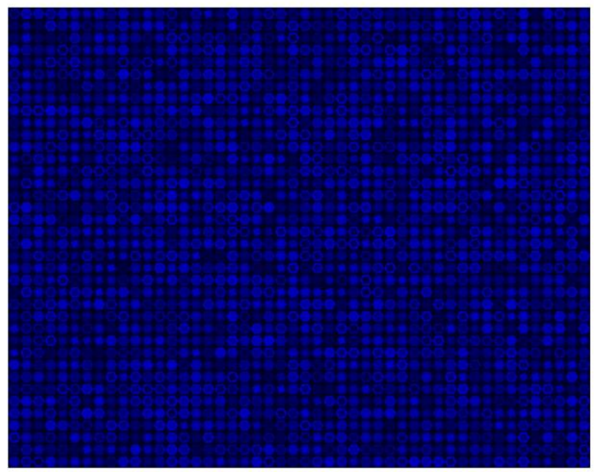

Figure 1-a. The primigenial universe.

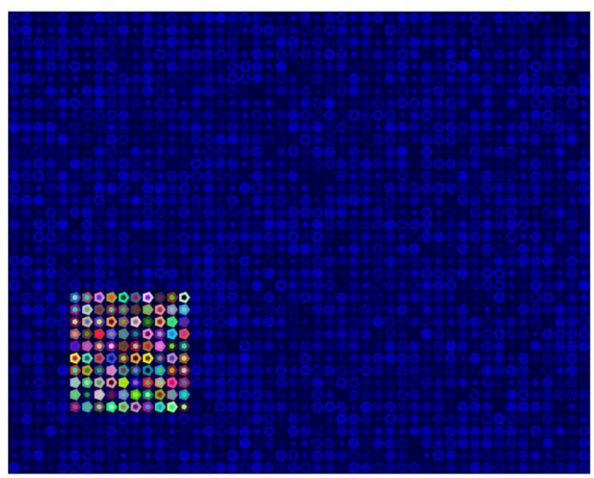

Figure 1-b. The Creation of the Earth.

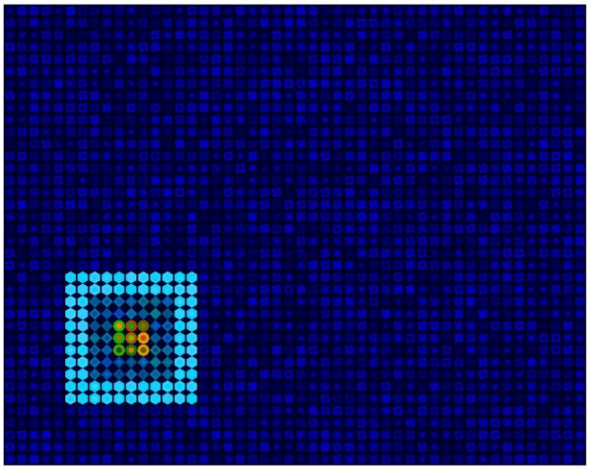

Figure 1-c. The Partition of the Earth into the earths, the sees, and the atmosphere.

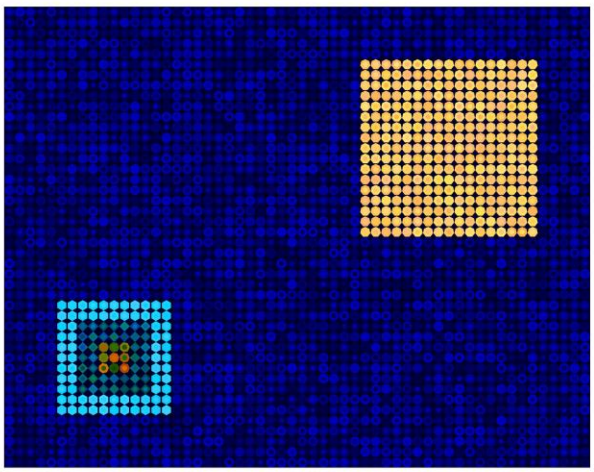

Figure 1-d. The creation of the Sun.

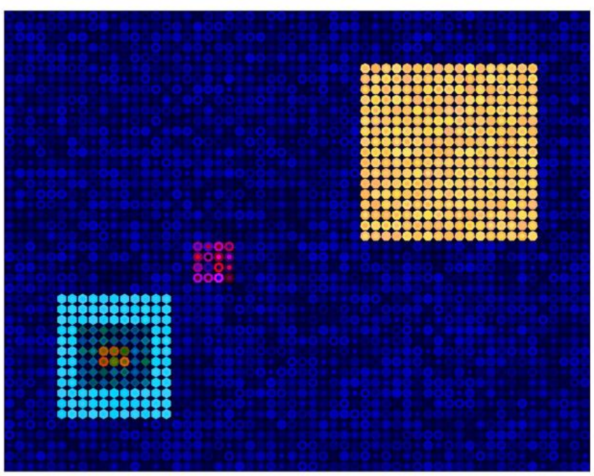

Figure 1-e. The Three-Body Universe: the Sun, the Earth, and the Moon.

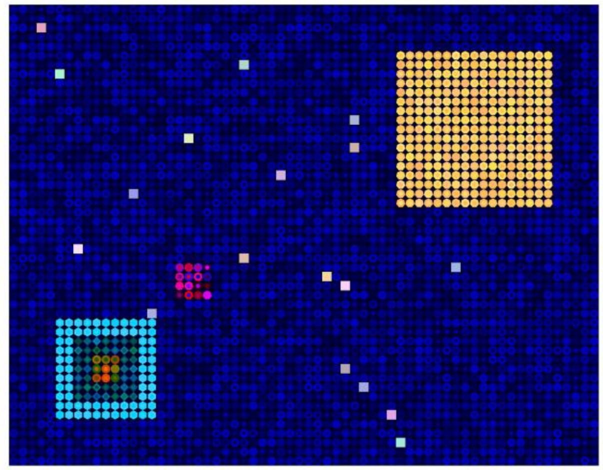

Figure 1-f. The Sun, the Earth, the Moon, and the Stars.

Figure 1. The sequence of the creation of the universe, day by day, may be assimilated to a sequence of refinements of the topology of the universe. At the end of each day the topology of the universe had gained some additional open sets.

Quantitative vs. qualitative, algebraic vs. geometric, approaches have always raised controversy in mathematics. In the next section, this manifestation of dualism is reviewed throughout the history of mathematics, to establish some historical references.

2 Qualitative vs. Quantitative Mathematics

Consistently with the presence of dualistic visions in different fields of knowledge, duality and dualism are also present in different forms in mathematics. So much so, that Courant and Robbins (Courant-Robbins 1996) started the opening essay of their famous book *What is Mathematics?* with these words:

"Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality. Though different traditions may emphasize different aspects, it is only the interplay of these antithetic forces and the struggle for their synthesis that constitute the life, usefulness, and supreme value of mathematical science."

The manifestations of dualism in mathematics are manifold and have been associated to social and philosophical cosmovisions along history. Thus, while practical needs promoted the development of arithmetic, algebra, and algorithmic in the Orient, and the start of its written recording around 2000 B.C., it was their afterwards diffusion to the Occident and their encounter with Greek philosophers and philosophy, what transformed mathematics, around the fifth and fourth centuries B.C., into a theoretical, contemplative, and axiomatic-deductive discipline for the next 2000 years. The geometric spirit of Greek mathematics was however challenged and superseded by the scientific revolution of the seventeenth century which propitiated the development of analytic geometry, differential and integral calculus, and transform differential equations, one of the most successful inventions of Newton, into the very language of expression of the laws of nature: “The laws of nature are expressed by differential equations”, in Newton’s own words (Arnol’d 1992). The dominance of the new, at times not so rigorous, quantitative approaches derived from differential calculus would last for next two hundred years, until the nineteenth century, when the global evaluation of higher education after the French revolution led to a revision of the foundations of mathematics, and particularly of differential and integral calculus. As a consequence of this review rigour was reintroduce into mathematics, a kind of internal discipline lasting until today. The development of mathematical analysis along the nineteenth century led to prove that most integrals are not explicitly solvable, what had a immediate catastrophic impact on differential equations theory because the insolvability of most integrals directly implies the insolvability of most differential equations. This event induced a schism in the study of differential equations, which bifurcated into the quantitative and the qualitative theories of differential equations. The former continued evolving along the traditional Newtonian computational approach, afterwards reinforced in the twentieth century by the introduction of digital numerical computation techniques. The originally visible face of the qualitative theory was the French mathematician Poincaré, who won the prize offered by the King of Sweden for solving the 3-body problem, but just by proving that such problem could not be explicitly solved (Hubbard & West 1991). Qualitative theory of differential equations is mostly based on geometry and topology, contrastingly to quantitative theory based on analysis and algebra.

With the brief and fast walk along the history of mathematics above we pretend to give some preliminary support to the statement that mathematics, like most disciplines and cultures, has a dualist nature which synergistically contrapose quantitative and qualitative arguments and approaches in the process of proposing and constructing models of the universe; no matter whether we understand the term universe in a physical or in a abstract sense. So, in our view, the even today widely accepted statement of Lord Rutherford (1871-1937): *“Qualitative is nothing but poor quantitative”* is neither fair nor appropriate, because these two words actually describe concepts belonging to different and complementary worlds, and therefore can not be straightforwardly compared. In what to this paper is concerned, the difference between qualitative and quantitative mathematics has to do with the kind of questions we wish to answer, and not with the amount of calculations required to answer any particular question. Qualitative mathematics deals with classification problems, whereas quantitative mathematics deals with computing particular solutions with the required exactitude.

The history of mathematics is a fascinating subject we cannot go deeper into in this paper. We refer the interested reader to (Courant & Robbins 1996) and (Stillwell 1989) for a global view of subject, to (Arnol’d 1992) and (Hubbard & West 1991) for the history of differential equations and dynamical systems, and to (Hirsch & Smale 1974) for the connection of the mathematical theory of differential equations and dynamical systems with applications in mechanics, circuits, population dynamics, and classical mechanics.

In next section we collect some basic results of topology and dynamical systems to give formal support to previous discussion and metaphors.

3 Basic Concepts of Topology

The main goal of qualitative mathematics is to solve classification problems, something that mathematicians approach using the concept of *equivalence relation*.

3.1 Equivalent Relations

Definition 1. A *relation* on a set A is a subset R of the Cartesian product $A \times A$, i.e. $R \subset A \times A$. For any two objects $x, y \in A$ we will say that *x is in the relation R to y*, denoted by xRy, if $(x, y) \in R \subset A \times A$.

Definition 2. The relation $R \subset A \times A$ is an *equivalence relation* on A if it has the following properties:

ER1. Reflexivity: xRx, for every $x \in A$. (11)

ER2. Symmetry: If xRy, then yRx. (12)

ER3. Transitivity: If xRy and yRz, then xRz. (13)

Definition 3. Given an equivalence relation R on a set A, each $x \in A$ determines a subset $E \subset A$, the *equivalence class* of x, defined as:

$$E = \{y \in A \mid yRx\}. \tag{14}$$

So, the equivalence class of x is the set of all those elements of A, which are equivalent to x under R.

Equivalence classes always have at least one element, and therefore are never empty. In fact, let E be the equivalence class of x under R. According to the definition above equivalence relations are reflexive, so xRx, then $x \in E$, hence $E \neq \Phi$.

Example 1. Second degree polynomials play an important role in the study of dynamical systems, in part because they can be exhaustively studied, both qualitative and quantitatively. Let $P^2 = \{p(\lambda) = \lambda^2 + a\,\lambda + b \mid a, b \in \mathbb{R}\}$ be the set of real second-degree polynomials. The application $I_P\colon P^2 \to \mathbb{R}^2$, $I_P(\lambda^2 + a\,\lambda + b) = (a, b)$, Figure 2, maps second-degree polynomials into points of the real plane, allowing to use $\mathbb{R}^2$ as a model of P^2.

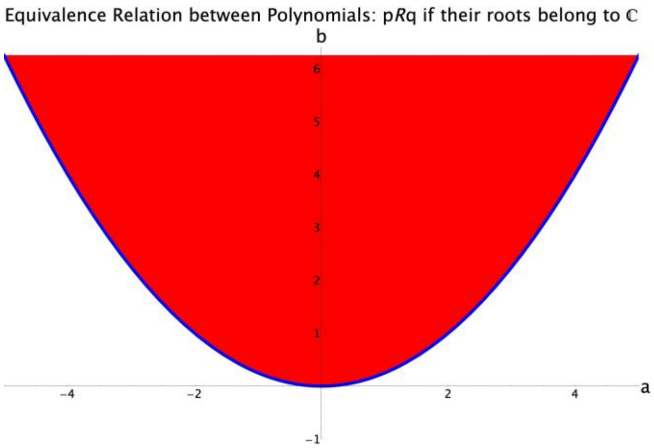

Figure 2. Equivalence classes. Ordered pairs $(a,b) \in \mathbb{R}^2$ model second-degree polynomials $\lambda^2 + a\,\lambda + b \in P^2$. The red zone, i.e., the subset $\{(a,b)|\ a^2 - 4b < 0\}$ of the real plane, models the subset of polynomials with complex conjugate roots. So, the **RZ** is the equivalence class of any second-degree polynomial with complex conjugate roots. The blue curve is the zero level curve of the discriminant function $\Delta(a,b) = a^2 - 4b$, which acts as the boundary of the equivalence class.

Two second degree polynomials $p, q \in P^2$ are R-related if their roots $\lambda_{1,2} = \frac{1}{2}(-a \pm \sqrt{a^2 - 4b}) \in \mathbb{C}$, which happens iff $a^2 - 4b < 0$. So, if $p(\lambda) \in P^2$ is a polynomial with complex conjugate roots, its equivalence class, under the equivalence relation R, is the red zone (**RZ**) of the space of polynomials in Figure 2.

That R is indeed an equivalence relation can be proved straightforwardly. Let $p_1, p_2, p_3 \in P^2$ with complex conjugate roots. Then $I_p(p_i) = (a_i, b_i) \in RZ$, for $i = 1, 2, 3$. (1) *Reflexivity*: If $(a_i, b_i) \in RZ$, $(a_i, b_i)R(a_i, b_i)$. (2) *Symmetry*: If $(a_i, b_i), (a_j, b_j) \in RZ$, then $(a_i, b_i)R(a_j, b_j)$ and $(a_j, b_j)R(a_i, b_i)$. (3) *Transitivity*: If $(a_1, b_1)R(a_2, b_2)$, $(a_i, b_i) \in RZ, i = 1, 2$. Analogously, if $(a_2, b_2)R(a_3, b_3)$, $(a_i, b_i) \in RZ, i = 2, 3$, then both $(a_1, b_1), (a_3, b_3) \in RZ$, hence $(a_1, b_1)R(a_3, b_3)$. ■

Classifications require of sharp cutting distinguishing criterions allowing separating objects possessing a particular property from objects not possessing such property. Equivalence relations play that role of sharp cutting knifes in mathematics, because the equivalence classes they generate are either equal or disjoint, and therefore objects are either equivalent or not equivalent, but not both.

Lemma 1. The equivalence classes of any two points $x, y \in A$ are either equal or disjoint.

Proof. Let E_x and E_y the equivalence classes of points $x, y \in A$, respectively. Then, $x \in E_x$ and $y \in E_y$. Let us suppose that $E_x \cap E_y \neq \Phi$, and let $z \in E_x \cap E_y$ be different to both $x, y \in A$. $z \in E_x \cap E_y$ implies that $z \in E_x$, then xRz, and $z \in E_y$, then yRz, hence zRy. Given that equivalence relations are transitive xRy. Let $v \in E_x$, such that $v \neq x$. If $v \in E_x$, then xRv, hence vRx. As R is transitive, vRx and xRy implies vRy, then for every $v \in E_x$, $v \in E_y$, and therefore $E_x \subset E_y$. Reversely, let $w \in E_y$, such that $w \neq y$. Then, yRw, and wRy. But yRx, then by transitivity xRw. Thus, for every $w \in E_y$, $w \in E_x$, then $E_y \subset E_x$. So, if $E_x \cap E_y \neq \Phi$, then $E_y = E_x$. ■

Lemma 2. Let R be an equivalence relation on A, and let $\mathcal{E}$ be the collection of all the equivalence classes associated to R. Then, the union of all elements of $\mathcal{E}$ equals all of A.

Proof. For every $x \in A$, x belongs to its equivalence class E_x. Consider another $y \in A$, which also belongs to its own equivalence class E_y. If yRx, then $E_y = E_x$, otherwise $E_y \cap E_x = \Phi$. So given that each $x \in A$ belongs to E_x, $A \subset \bigcup_{x \in A} E_x$. ■

Definition 4. A *partition* of a set A is a collection of disjoint subsets of A whose union is all of A.

Example 2. Within the same context of Example 1, we will denote by $P^2 = \{p(\lambda) = \lambda^2 + a\,\lambda + b \mid a, b \in \mathbb{R}\}$ the set of real second degree polynomials, which we identify with the real plane through the application $I_P: P^2 \to \mathbb{R}^2$, $I_P(\lambda^2 + a\,\lambda + b) = (a, b)$. The red zone (**RZ**) of Figure 2 is the equivalence class of the polynomials with complex conjugate roots. The blue zone (**BZ**) is the equivalence class of all polynomials with real roots. As it is well known, roots are either complex conjugate or real, and there do not exist any kind of mixed second order polynomials with one real and one complex root. This implies that the subsets of second degree polynomial with real and complex conjugate roots are disjoint, and so are their equivalence classes, as shown in Figure 2, where the **RZ** correspond to polynomials with negative discriminants, and the **BZ** to polynomials with non-negative discriminants. The boundary between the **RZ** and the **BZ** is precisely the zero-level curve of the discriminant function $\Delta(a,b) = a^2 - 4\,b$, associated to polynomials with real double roots. So, $\boldsymbol{RZ} \cap \boldsymbol{BZ} = \Phi$, and $\boldsymbol{RZ} \cup \boldsymbol{BZ} = \mathbb{R}^2 \approx F^2$. ■

The two lemmas above just say that on the one hand equivalence classes are either disjoint or equal, and on the other hand the union of all equivalence classes equals all of A, which is the same to say that each equivalence relation generates a partition of A, allowing to decompose the set A into the disjoint union of the equivalence classes of the elements of A under the equivalence relation. The following lemma just says that studying the partitions of a set A, or studying the equivalence relations on A is exactly the same. The proof of this lemma is of great potential practical application, because it suggests how to design partitions, then equivalence relations, for practical purposes.

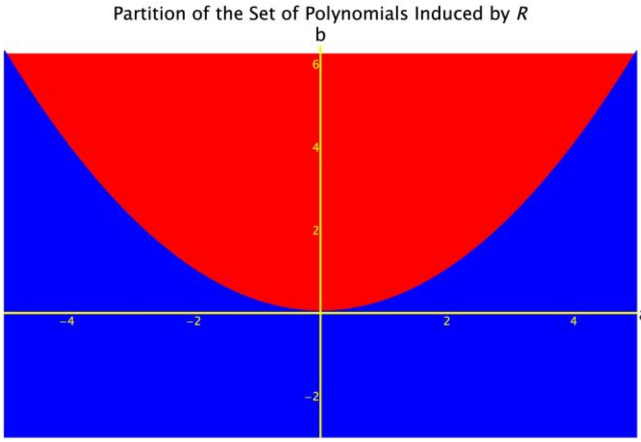

Figure 2. The partition of the space of second-degree polynomials induced by the equivalence relation R: For every pair of polynomial $p, q \in P^2$, pRq if their roots are complex conjugate. In Example 1 it was already shown that the red zone (**R**Z) is the equivalence class of the polynomials with complex conjugate roots. The blue zone (**BZ**) is the equivalence class of all polynomials with real roots. As expected $\boldsymbol{RZ} \cup \boldsymbol{BZ} = \mathbb{R}^2 \approx P^2$. ■

Lemma 3. Given any partition $\mathfrak{D}$ of A, there exist exactly one equivalence relation on A from which it is derived.

Proof. We will prove first that partition $\mathfrak{D}$ comes from an equivalence relation D on A. For every $x, y \in A, xDy$ if both x and y belong to the same element of $\mathfrak{D}$. We must prove that D is an equivalence relation. D is symmetric because for every $x \in A$, x and x belongs to the same element of $\mathfrak{D}$. Partition $\mathfrak{D}$ covers all of A. Let $x, y \in A$. Then $x \in X \subset A$, $y \in Y \subset A$, where X and Y are elements of the partition $\mathfrak{D}$. Given that the elements of $\mathfrak{D}$ are disjoint, either $X = Y$ or $X \cap Y = \Phi$. If $X = Y$, both relations xDy and yDx hold. The elements of partition $\mathfrak{D}$ are disjoint. Let T be an element of $\mathfrak{D}$, such that $x, y, z \in T$. Then, xDy, yDz, and xDz hold simultaneously. So, the binary relation D do is an equivalence relation. We finally observe that every single $x \in A$ belongs to a unique element E_x of partition $\mathfrak{D}$. Moreover, every $x \in A$ also belongs to its own equivalence class D_x, which intersects no other equivalence class of the equivalence relation D. So, $x \in E_x \cap D_x \neq \Phi$, then necessarily $E_x = D_x$.

We must prove now that the equivalence relation D is unique. Suppose there exist two equivalence relation D_1 and D_2 which equivalence classes are the same collection of disjoint elements of the partition $\mathfrak{D}$. Let E_1 be the equivalence class of x under D_1, and let E_2 be the equivalence class of x under D_2. Then E_1 is the unique element E of partition $\mathfrak{D}$ containing x. Analogously, E_2 is the unique element E of partition $\mathfrak{D}$ containing x. $E_1 = \{y \in A \mid yD_1x\}$, and $E_2 = \{y \in A \mid yD_2x\}$. Then $E_1 = E = E_2$, which implies that $D_1 = D_2$. ■

Example 3. Partition in Example 2 allows us separating second-degree polynomials F^2 into two disjoints classes, i.e., $RZ \cup BZ = \mathbb{R}^2 \approx F^2$, where RZ and BZ gather together the polynomials with complex conjugate and real roots, respectively. We proceed now to refine that partition, to get a finer classification of second-order polynomials.

Let $(a, b) \in F^2$ be a second-degree polynomial with roots $\lambda_{1,2} = \frac{1}{2}(-a \pm \sqrt{a^2 - 4b})$. If $b < 0$, the discriminant $\Delta(a, b) = a^2 - 4b > 0$, and $\lambda_{1,2} \in \mathbb{R}$. Yet, for $b < 0$, $\Delta(a, b) \geq a^2$, and then one of the roots in positive and the other one negative. When $b > 0$, $\Delta(a, b) \leq a^2$; then, if $a > 0$ both roots are either real and negative, or complex conjugate with negative real part, whereas if $a < 0$, both roots are real and positive, or complex conjugate with positive real part. ■

The classification of second-degree polynomials according to the structure of their roots, summarized in the classification map of Figure 3, plays a very important role in the theory of dynamical systems (Hirsch-Smale 1974, Arnol'd 1992), as we shall see below. By the time being let suffice it to say that Figure 3 clearly shows that second-degree polynomials are structurally stable with respect to small perturbations in their parameters. In fact, if (a, b) is an arbitrary polynomial belonging to any one of the equivalence classes of the partition in Figure 3, all nearby polynomials (a', b') also belong to the same equivalence class, and therefore have the same root structure of polynomial (a, b). Structural stability, or robustness, of polynomials is a fundamental and highly appreciated feature in most engineering applications. Figure 3 also clearly shows that the polynomials located at the boundaries between equivalence classes are not structurally stable, because small perturbations of their parameters might completely alter the structure of their roots.

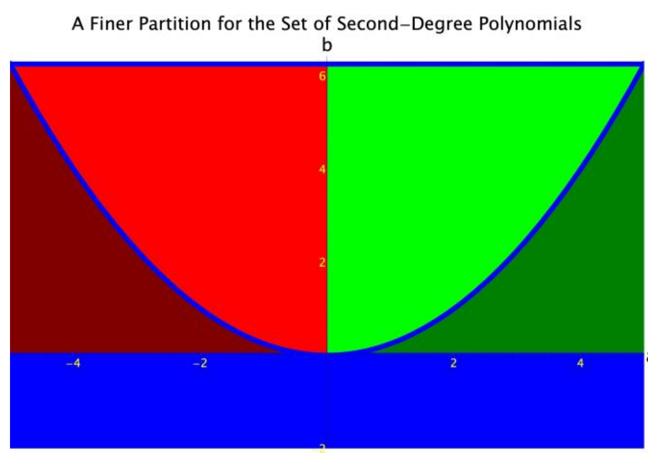

Figure 3. A finer partition for the set of second-degree polynomials. In this refined partition the blue zone corresponds to polynomials with real roots of opposite sign. The polynomials of the dark (light) red zone are real (complex conjugate) and positive (with positive real part), while the polynomials of the dark (light) green zone are real (complex conjugate) and negative (with negative real part).

3.2 Topological Spaces and Quotient Spaces

Definition 5. A *topology* on a set X is a collection $\mathfrak{J}$ of subsets $U \subset X$ such that:
T1. X and Φ belong to $\mathfrak{J}$. (15)
T2. An arbitrary union of elements of $\mathfrak{J}$ is in $\mathfrak{J}$. (16)
T3. Any finite intersection of elements of $\mathfrak{J}$ is in $\mathfrak{J}$. (17)
The pair (X, $\mathfrak{J}$) consisting of the set X equiped with the topology $\mathfrak{J}$ is called a *topological space*. The elements $U \in \mathfrak{J}$ of the topology $\mathfrak{J}$ are called *open set*.

Example 4. In Section 1.2 above we showed that Taoist and Platonist cosmovisions lead to the topologies $\mathcal{T} = \{T, X, Y, \Phi\}$, with $X \bigcup Y = T$ and $X \bigcap Y = \Phi$, and $\mathcal{P} = \{P, I, R, \Phi\}$, with $I \bigcup R = P$, $I \bigcap R = \Phi$, on the Taoist and Platonist universes T and P, respectively. Then, {X, Y} and {I, R} are partitions of T and P, respectively, and the pairs $(T, \mathcal{T})$ and $(P, \mathcal{P})$ qualify to be topological spaces. ∎

Let X be a set, and U, V two subsets of X, such that $U \bigcup V = X$ and $U \bigcap V = \Phi$. The partition D = {U, V} of X and the topology $\mathfrak{D} = \{X, U, V, \Phi\}$ on X do not have proper names in mathematics; yet, withing the framework of the present work, we will called D, $\mathfrak{D}$, and $(D, \mathfrak{D})$ the *dualist partition*, the *dualist topology*, and the dualist topological space $(D, \mathfrak{D})$, respectively.

Definition 6. Let $\mathfrak{J}$ and $\mathfrak{J}'$ be two topologies on X. If $\mathfrak{J} \subset \mathfrak{J}'$, we say that $\mathfrak{J}'$ *is finer than* $\mathfrak{J}$, or alternatively we say that $\mathfrak{J}$ *is coarser than* $\mathfrak{J}'$.

Example 5. In Case Study 2, we considered a system X consisting of a seller, S, and the set of his clients, C. Given that $\{S\} \bigcup C = X$, $\{S\} \bigcap C = \Phi$, the mathematical objects $D = \{S, C\}$, $\mathfrak{D} = \{X, S, C, \Phi\}$, and $(D, \mathfrak{D})$ are a dualist partition of X, a dualist topology on X, and a dualist topological space, respectively. However, the mathematical structure of the dualist topological space $(D, \mathfrak{D})$ rest upon a colective cultural myth, namely the existence and appropriate functioning of power, telecommunication, and data networks, what in developed countries is taken for granted. When a blackout occurs, the set of clients C suffers a dualist partition generating a new finer partition $D_1 = \{S, C, M, E\}$, and an associated finer topology $\mathfrak{D}_1 = \{X, S, C, M, E, \Phi\}$ on X, where M is the subset of clients who possess cash, and E is the subset of clients who can only pay with a debit card. Subsets M and E are such that $\{M\} \bigcup E = C$, $\{M\} \bigcap E = \Phi$. In turn, the subset E also suffers a dualist partition into the disjoint subsets of the old trust clients F, and the unknown clients G, which generates an even finer partition $D_2 = \{S, C, M, E, F, G\}$, and the associated finer topology $\mathfrak{D}_2 = \{X, S, C, M, E, F, G, \Phi\}$ on X. ∎

Quotient sets may be naturally equipped with quotient topologies, transforming them into quotient spaces, which among other things allows defining continuous functions on them (Dugundji 1976, Kelley 1955, Munkres 1972).

Definition 7. Let X and Y be topological spaces, and let $p: X \to Y$ be a surjective map. The map p is called a *quotient map*, provided a subset U of Y is open in Y iff $p^{-1}(U)$ is open in X.

Definition 8. Let X be a topological space. Let A be a set and $p: X \to A$ a surjective map. Then, there exists exactly one topology $\mathfrak{J}$ on A, relative to which p is a quotient map. This topology is called the *quotient topology* induced by p.

Definition 9. Let X be a topological space, and let X^* be a partition of X into disjoint subsets whose union is all of X. Let $p: X \to X^*$ be a surjective map sending each point of X into the element of partition X^* containing it. In the quotient topology induced by p, the space X^* is called a *quotient space* of X.

Example 6. Let F^2 be the set of the second-degree polynomials. Let R be the equivalence relation generating the partition F^2/R of F^2, described by the classification map of the polynomials of Figure 3, and consisting of five equivalence classes: the blue zone **BZ**, the dark red zone **DRZ**, the light red zone **LRZ**, the dark green zone **DGZ**, and the light green zone **LGZ**, where

$$\begin{aligned}
\boldsymbol{BZ} &= \{q \in F^2 \mid p \text{ has real roots of opposite signs}\} \\
\boldsymbol{DRZ} &= \{q \in F^2 \mid p \text{ has real positive roots}\} \\
\boldsymbol{LRZ} &= \{q \in F^2 \mid \text{roots in } \mathbb{C} \text{ with positive real part}\} \\
\boldsymbol{DGZ} &= \{q \in F^2 \mid p \text{ has real negative roots }\} \\
\boldsymbol{LGZ} &= \{q \in F^2 \mid \text{roots in } \mathbb{C} \text{ with negative real part}\}.
\end{aligned}$$

Let $p: F^2 \to F^2/R$ be the identification given by $p(r) = [r]$, where [r] is the equivalence class of r under de equivalence relation R. The projection p is surjective. In the quotient topology $\mathfrak{J}$ induced by p on F^2/R, the topological space $(F^2/R,\ \mathfrak{J})$ is the quotient space of F^2. So, $\mathfrak{B} = \{\boldsymbol{BZ}, \boldsymbol{DRZ}, \boldsymbol{LRZ}, \boldsymbol{DGZ}, \boldsymbol{LGZ}\}$ is a basis for the topology $\mathfrak{J}$ of the quotient space F^2/R, consisting of F^2 and Φ, the five elements of $\mathfrak{B}$, and all their possible unions. Intersections of the elements of B are not even mentioned because B is a partition of F^2 ∎

The Contents of Part II-B of present paper will be:

4 Conclusions

4.1 Dualism, Multiculturalism, and Topologies

Metaphysical arguments aside, dualism has historically proved to be a very efficient tool to describe and model all those systems whose dynamics essentially depends on two actors. The success of dualism is topological associated to equipping the supporting space X of the studied phenomena with a topology $\mathfrak{J} = \{A, C, X, \Phi\}$ consisting of two open sets A and C, such that $A \cup C = X$, and $A \cap C = \Phi$. The disjoint partition $\{A, C\}$ is the simple most topological structure capable to support the "dialog" between representatives of A and C, as equivalence classes, where a "dialog" is nothing but the most primitive formulation of what in contemporary mathematical language is called a second order dynamical system. Dialog pursues agreements, that is to say, permanent settle downs, i.e., equilibrium points, among actors in conflict. In modern notation, the first model for the dynamics of a second-order system is a linear second-order dynamical system, whose study sooner or later converges to the exhaustive analysis of its characteristic polynomial, which eventually leads to construct, either implicitly or explicitly, the topological classification map of the system, describing all possible different qualitative dynamics.

It is crystal clear that not every phenomenon admits being modeled as a second-order dynamical system, what leads to introduce bigger topologies with more open sets, higher order associated dynamical systems to model the dynamics of the systems, and higher degree polynomials to generate all the fundamental solutions of the systems. The complexity of this problem increases enormously with the number of equivalence classes of the systems and the order of the associated dynamical system. This is the world of multiculturality, where single cultures coexist but do not intermingle. Topological partitions lead to quotient topologies and quotient spaces, but impede interculturality.

For interculturality to emerge, the topology of X must contain open sets with non-trivial intersections. This would also obviously lead to richer "dialogs" and "forms of dialog", or equivalently, to more complex dynamical systems, necessarily defined on topological spaces with richer topological structures. So, at least from the strictly mathematical point of view, it is natural to expect that multicultural structures would be preferred to intercultural structures, even if it were just for complexity reasons, not to mention the required levels of intellectual sophistication required to approach interculturalism interculturally. This is a very interesting topic deserving deeper effort and study.

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